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Laplace transforms

Definition of Laplace transforms:
 It is sort of like averaging a function over time while introducing the variable s :

$$f(t) \xrightarrow{\mathcal{L}} f(s) = \int_0^{\infty} e^{-st} f(t) dt$$

Why use them? They have interesting properties!

$$f(t) \xrightarrow{\mathcal{L}} f(s)$$

$$\frac{df(t)}{dt} \xrightarrow{\mathcal{L}} sf(s) - f(t=0, \dots)$$

This is the most important property!

$$\frac{\partial f(t, x)}{\partial x} \xrightarrow{\mathcal{L}} \frac{df(s)}{dx}$$

It allows us to eliminate time differentials. This can make very hard problems much easier!

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Laplace transforms

The general procedure for a “hard” problem:

1. Start with a hard problem
2. Do a Laplace transform: get an easy problem
3. Solve the easy problem
4. Do the reverse Laplace transform → this is usually the hard part!

For example:

$$1. \frac{\partial C(\theta, Z)}{\partial \theta} + \frac{\partial C(\theta, Z)}{\partial Z} = 0 \quad \text{The hard problem!}$$

$$\downarrow \mathcal{L}$$

$$2. sC(s, Z) - C(\theta = 0, Z) + \frac{dC(s, Z)}{dZ} = 0 \quad \text{An easy problem!}$$

$$3. \text{ For a Dirac: } C(s, Z)$$

$$\downarrow \mathcal{L}^{-1}$$

$$4. C(\theta, Z) = f(t, z)$$